# Hyper Geometric Function of Gaussian and Some Applications

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## Abstract:

Recent work has investigated a few generalisations of well-known special functions (such as the Gamma function, Beta function, Gauss hyper geometric function, etc.). In addition, the Pfaff-Saalschütz theorem is given as a special case from it, some new integrals using the generalised Gauss hyper geometric functions are obtained, and many significant results are noted. The main goal of this paper is to express explicitly the generalisation of the classical generalised hyper geometric function pdf in terms of the classical generalised hyper geometric function itself.

**Keywords: Hyper Geometric, Gamma Function**

**Introduction:** One of the least complex yet significant uncommon functions is the Gamma function, defined by

$$\Gamma(z) = \begin{cases} \int_0^\infty e^{-1} t^{z-1} dt & ; Re(z) > 0 \\ \frac{\Gamma(z+1)}{z} < 0 & ; z \neq -1, -2, -3 \ldots \end{cases}$$

Truth be told, the Gamma function Γ is a speculation of the factorial function z!, the space of positive whole numbers to the area of every single genuine number expect as 0,- 1, - 2,- 3…
To present a systematic study of elementary functions and the geometric series, C.F. Gauss, in 1812, introduced the series,

$$\sum_n^\infty \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} = 1 + \frac{a.b}{1.c} z + \frac{a(a+1)b(b+1)}{1.2.c\,(c+1)} z^2 + \ldots\ldots$$

This is of great importance to mathematicians. This series in known as the Gauss series and represented by the symbol $_2F_1$ (a, b; c; z), this function satisfies a second order linear differential equation with three singularities 0, 1, ∞ the three components a, b and c are depicted as the parameters and z is known as the variable of the arrangement. Every one of the four of these amounts might be any genuine numbers, genuine or complex aside from c is a non-positive whole number. When all is said in done, if both of the numerator parameters is a negative whole number, the arrangement ends.
The Gauss Hyper geometric function $_2F_1$ (a, b; c; z) is defined in the unit disk as the sum of the Hyper geometric series and represented as follows:

$$_2F_1\,(a, b; c; z) = {_2F_1} \left| \begin{matrix} a, b \\ c \end{matrix} ; z \right| = \sum_{k=0}^\infty \frac{(a)_k (b)_k}{(c)_k} \frac{z^k}{k!}$$

Where
$|z| < 1; a, b \in C;$ c $\in C \neq z_0^-$
and $(a_k)$ is the Pochhammer Symbol.

$$(a)_k = \begin{cases} 1 & if\ k = 0 \\ a(a+1)..(a+k-1), & if\ k = 1,2,.. \end{cases}$$

The series above is absolutely convergent for $|z| < 1$ and for $|z| = 1$ when Re (c-a - b) > 0; while it is conditionally convergent for $|z| = 1$ $(z \neq 1)$ .
If-1<Re(c-a-b) <0
$2F_1$(a, b; c; z) The unusual case contains Legendre, defective beta functions, the absolute first and second form of elliptic equation, and a significant number of old style symmetrical polynomials.
If, c$\notin$ $z_0^-$, the $_2F_1$(a, b; c; z) has another integral representation in terms of the Mellin- Barnes contour integral
The confluent Hyper geometric Kummer function is defined by the series

$$\phi(a; b; z) = 1F_1 \left| \begin{matrix} a \\ c \end{matrix} ; z \right| = \sum_{k=0}^\infty \frac{(a)_k}{(c)_k} \frac{z^k}{k!}$$

Where z, a ∈ ℂ ≠ ℤ; but, in contrast to the Hypergeometric series, this series is convergent for any The contour integral representation of series is defined by:

$\phi(a;b;z) = \frac{1}{2\pi i} \frac{\Gamma(c)}{\Gamma(a)} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\Gamma(a+s)}{\Gamma(c+s)} \Gamma(-s)(-z)^s \, ds,$

Where $|\arg(-z) < \pi|$ and the path of integration separates all the poles of $\Gamma(a+s)$ to the left and the poles of $\Gamma(-s)$ to the right.

The Gauss Hyper geometric arrangement and the Kummer Hyper geometric arrangement are stretched out to the summed up Hyper geometric arrangement. In the Gaussian Hyper geometric series ${}_2F_1$ (a, b; c; z), Two parameters of the numerator a, b and c are visible. Any arbitrary number of numerator and denominator parameters is introduced to generalize the sequence naturally. The consequence is defined by

$p\,Fq \begin{bmatrix} \alpha_1, \ldots \alpha_2 \\ \beta_1, \ldots, \beta_2 \end{bmatrix} = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \ldots (\alpha_p)_n}{(\beta_1)_n \cdots (\beta_q)_n} \frac{z^n}{n!}$

Where $(\lambda)n$ is the Pochhammer image characterized? The scheme is known as the Gaus simplified scheme, or simply the Hyper geometric scheme simplified. p and q here are likely to have entire or null numbers, so we consider the vector z to be the parameters of the numerator, $\alpha_1, \ldots \alpha_p$ and the denominator parameters $\beta_1, \ldots, \beta_q$ take on complex values, provided that

$$\beta_j = 0, -1, -2, \ldots; (j = 1, \ldots, q)$$

Assuming that none of the numerator parameters is zero or a negative number (generally the subject of union won't emerge), and with the standard confinement the ${}_pF_q$ series in

1. Converges for $|z| < \infty \; if \; p \le q,$
2. Converges for $|z| < 1 \; if \; p = q + 1$ and

Diverges for all z, $|z| \neq 0 \; if \; p > q + 1$

Furthermore, if

$\omega = \sum_{j=1}^{q} \beta_j - \sum_{j=1}^{p} \alpha_j,$

It is known that the pFq series with p = q + 1 is;

1) absolutely convergent for $|z| = 1 \; if \; Re(\omega) > 0$
2) conditionally convergent for $|z| = 1, |z| \neq 1, if -1 < Re(\omega) \le 0$

Divergent for $|z| = 1 \; if \; Re(\omega) \le -1$

An important special case of the series is the Kummer Hyper geometric series ${}_1F_1$ (a; c; z) defined in which case p = q = 1, since,

${}_1F_1(a; c; z) = \lim_{|\mu| \to \infty} \left(a; b; c; \frac{z}{b}\right)$

The Mellin –Barnes counter integral representation of ${}_pF_q$ series is given by

${}_pF_q(a_1, \ldots, a_p; b_1, \ldots b_q; z) = {}_pF_q \begin{bmatrix} a_1, \ldots q_p \\ b_1, \ldots, b_q \end{bmatrix}; z$

An interesting further generalization of the series ${}_pF_q$ is due to Fox and Wright; who studied the asymptotic expansion of the generalized Hyper geometric function defined by,

${}_p\Psi_q \left| \begin{matrix} (\alpha_1, A_1), \ldots, (\alpha_p, A_p); \\ (\beta_1, B_1), \ldots, (\beta_q, B_q); \end{matrix} z \right| = \sum_{n=0}^{\infty} \frac{\Pi_{j=1}^{p} \Gamma(\alpha_j + A_j n)}{\Pi_{j=1}^{q} \Gamma(\beta_j + B_j n)} . \frac{z^n}{n!}$

Where the coefficient $A_1, \ldots A_p$ and $B_1, \ldots, B_q$ are positive real numbers such that

$\sum_{j=1}^{q} B_j - \sum_{j=1}^{p} A_j \ge 0$

If we take $A_j = 1, \ldots, p$ and $B_j = 1; j = 1, \ldots, q$ the relation becomes,

${}_p\Psi_q \left| \begin{matrix} (\alpha_1, 1), \ldots, (\alpha_p, 1); \\ (\beta_1, 1), \ldots, (\beta_q, 1); \end{matrix} z \right| = \frac{\Pi_{j=1}^{p} \Gamma(\alpha_1)}{\Pi_{j=1}^{q} \Gamma(\beta_1)} {}_pF_q \begin{bmatrix} \alpha_1, \ldots \alpha_p; \\ \beta_1, \ldots, \beta_q \end{bmatrix}; z$

Where ${}_pF_q$ (z) is the generalized Hyper geometric function defined.

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