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Some Identities on Generalized Fibonacci Sequence and Pell-Lucas Sequence

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ABSTRACT

The eminent Fibonacci (and Lucas) sequence is one of the sequences of positive entire numbers that have been focused on more than a seriously extended period of time. A couple of makers consider a couple of properties for the k-Fibonacci numbers got from simple grid variable based math and its characters including delivering capacity and distinctness properties.

The sequence of Pell numbers is other sequence of numbers that is portrayed by the recursive sequence with the hidden conditions. This sequence has been thought of and a part of its fundamental properties are known. We track down the cross section technique for delivering this sequence and comparable framework for the Fibonacci and Pell sequences. On occasion, in the composition, are seen as various sequences explicitly, Pell-Lucas and Changed Pell sequences and besides new structures which rely upon these sequences as well as that they have the making cross sections.

KEYWORDS: Fibonacci, Lucas, Sequence

INTRODUCTION

We consider a hypothesis of the Fibonacci sequence involving a recurrent association of higher solicitation. Specifically, they consider, for a number k > 2, the k-summarized Fibonacci sequence which looks like the Fibonacci sequence yet starting with 0, 0, . . . , 0, 1 (an amount of k terms) and each term in this manner is how much the k going before terms. A Binet-style recipe that can be used to make the k-summarized Fibonacci numbers and captivating calculating properties of these numbers is given.(Koshy, 2011)[1]

In like manner, there is the Pell sequence, which is fundamentally pretty much as critical as the Fibonacci sequence. The Pell sequence (Pn)∞ n=0 is portrayed by the recurrent Pn = 2Pn−1 + Pn−2 for all n > 2 with P0 = 0 and P1 = 1.

The Binet's condition is in like manner prominent for a couple of these sequences. At times a couple of fundamental properties come from this recipe.

As a peculiarity for 1, we get the silver extent which is associated with the Pell number sequence. Silver extent is the limiting extent of constant Pell numbers. At times a couple of fundamental properties come from the Binet's recipe.

There are various number sequences which are used in basically every field of current sciences. The Fibonacci sequence has been summarized in various ways, a few by saving the fundamental conditions, and others by safeguarding the recurrent association. (Bilgici, 2014)[2]

Kili¸c presented a couple of relations including the normal Fibonacci and k-Pell numbers showing the way that the k-Pell numbers can be imparted as the summation of the standard Fibonacci numbers. The makers gave one more hypothesis of the Pell numbers in cross section depiction and exhibited the way that the measures of the summarized Pell numbers could be resolved clearly using this depiction.

A couple of captivating characters including the Fibonacci and summarized Pell numbers are similarly inferred and a couple of eminent properties of P (2) are summarized to the sequence P (k) .it is well known Generalized Fibonacci sequence $\{Un\}$,

$Un+1 = pUn + qUn-1$, $U0 = 0$ and $U1 = 1$,

and generalized Lucas sequence $\{Vn\}$ are defined by

$Vn+1 = pVn + qVn-1$, $V0 = 2$ and $V1$ = p

[1] Koshy, Fibonacci and Lucas Numbers with Applications, Wiley-Interscience, New York, NY, USA, 2011

[2] Bilgici, New generalizations of Fibonacci and Lucas sequences, Appl. Math. Sc., 8(29)(2014), 1429–1437.

where p and q are nonzero real numbers and $n \geq 1$. For $p = q = 1$, we have classical Fibonacci and Lucas sequences. For $p = 2$, $q = 1$, we have Pell and PellLucas sequences. For detailed information about Fibonacci and Lucas numbers. (Dasdemir, 2011)[3]

These new hypotheses integrate every one of the more amazing relationship with hypotheses of Fibonacci numbers. We give a couple of properties of these new hypotheses and get a couple of relations between the summarized demand k Lucas numbers and the summarized demand k Fibonacci numbers.

There are various kinds of hypotheses of Fibonacci and Lucas numbers. Study described Fibonacci k-numbers {Fk,n}, for k ≥ 1, Fk,0 = 0, Fk,1 = 1 and Fk,n = kFk,n−1 + Fk,n−2 for n ≥ 2. It is easy to see that for k = 1 Fibonacci k-sequence is reduced to the ordinary Fibonacci sequence and for k = 2, it is diminished to the standard Pell sequence. A couple of makers described a summarized Fibonacci sequence as Fn+1 = pFn + qFn−1, where p and q are standard numbers, which go about as the control limits.

Summarized Fibonacci polynomials are diminished, by suitable substitutions, to Fibonacci k-numbers {Fk,n} , summarized Fibonacci sequence, Fibonacci pnumbers, summarized Pell (p, i)−numbers and bivariate Fibonacci p-polynomials, etc.(Kose, 2012)[4]

Additionally, Summarized Lucas polynomials are diminished, by suitable substitutions, to Lucas p numbers, m-increase of the Lucas p-numbers, bivariate Lucas p-polynomials, Lucas p-polynomials, Lucas polynomials, Lucas pnumbers, Lucas numbers, bivariate Pell-Lucas p-polynomials, bivariate PellLucas polynomials.

SOME IDENTITIES ON GENERALIZED FIBONACCI SEQUENCE AND PELL-LUCAS SEQUENCE

The Fibonacci numbers Fn are the details of the sequence {0,1,1,2,3, 5,8 * * *} wherein each term is the amount of the two past terms starting with the underlying qualities F0 = 0 and F1 = 1. Likewise the proportion of two successive Fibonacci numbers unites to the Brilliant mean, 0 = (1 + [square base of 5)]/2.(Taskara, 2012) [5]

The Fibonacci numbers and Brilliant mean track down various applications in present day science and have been widely utilized in number hypothesis, applied math, physical science, software engineering, and science. The well-known Fibonacci sequence is defined as

[F0] = 0, [F1] = 1,

[Fn] = [F_{n-1}] + [F_{n-2}] for n [greater than or equal to] 2.

In a similar way, Lucas sequence is defined as

[L0] = 2, [L_1] = 1,

[Ln] = [L_{n-1}] + [L_{n-2}] for n [greater than or equal to] 2.

The second order Fibonacci sequence has been generalized in several ways. Some authors have preserved the recurrence relation and altered the first two terms of the sequence while others have preserved the first two terms of the sequence and altered the recurrence relation slightly. (Koshy, 2014)[6]

The k-Fibonacci sequence depends only on one integer parameter k and is defined as follows:

[$F_{k,0}$] = 0, [$F_{k,1}$] = 1,

[$F_{k,n+1}$] = k[$F_{k,n}$] + [$F_{k,n-1}$], where n [greater than or equal to] 1, k [greater than or equal to] 1.

The first few terms of this sequence are

{0, 1, k, [k^2] + 1,[k^2] + 2 * * *}.

[3] Dasdemir, On the Pell, Pell-Lucas and modified Pell numbers by matrix method, Appl. Math. Sci., 5(64)(2011), 3173–3181

[4] Kose, "On the sequence related to Lucas numbers and its properties," Mathematica Aeterna, vol. 2, no. 1, pp. 63-75, 2012.

[5] Taskara, On the (s, t)-Pell and (s, t)-Pell-Lucas sequences and their matrix represantations, Appl. Math. Lett., 25(2012), 1554–1559.

[6] Koshy, Pell and Pell-Lucas numbers with applications, Springer, Berlin, 2014

The particular cases of the k-Fibonacci sequence are as follows.
If k = 1, the classical Fibonacci sequence is obtained:
[F0] = 0, [F1] = 1,
[F_{n+1}] = [Fn] + [F_{n-1}] for n [greater than or equal to] 1,
[{[Fn]}. n[member of]N] = {0, 1, 1, 2, 3, 5, 8 *** }.
If k = 2, the Pell sequence is obtained:
[P0] = 0, P = 1, [P_{n+1}] = 2[Pn] + [P_{n-1}] for n [greater than or equal to] 1,
[{[Pn]}. n[member of]N] = {0, 1, 2, 5, 12, 29, 70 *** }.
Motivated by the study of k-Fibonacci numbers , the k-Lucas numbers have been defined in a similar fashion as
[$K_{k,0}$] = 2, [$L_{k,1}$] = k,
[$L_{k,n+1}$] = k[$L_{k,n}$] + [$L_{k,n-1}$], where n [greater than or equal to] 1, k [greater than or equal to] 1.
The first few terms of this sequence are
{2, k, [k^2] + 2, [k^3] + 3 *** }.
The particular cases of the k-Lucas sequence are as follows.
If k = 1, the classical Lucas sequence is obtained:
{2, 1, 3, 4, 7, 11 , 18 *** }.
If k = 2, the Pell-Lucas sequence is obtained:
{2, 2, 6, 14 , 34, 82 *** }.
In the 19th century, the French mathematician Binet devised two remarkable analytical formulas for the Fibonacci and Lucas numbers. The same idea has been used to develop Binet formulas for other recursive sequences as well. (Bolat, 2012)[7]
The well known Binet's formulas for k-Fibonacci numbers and k-Lucas numbers, are given by
[Fk,n] = [r1n] - [r2n]/[r1n] - [r2n],
[L k,n] = [r1n] + [r2n], (11)
where [r1], [r2] are roots of characteristic equation
[r2] - kr - 1 = 0,
which are given by
[r1] = k + [square root of [k2] + r]/2, [r2] = k - [square root of [k^2] + 4]/2.
We also note that
[r1] + [r2] = k,
[r1] [r2] = -1,
[r1] - [r2] = [square root of [k2] + r].
There are endless direct as well as summarized characters open in the Fibonacci related writing in various designs.. The k-Fibonacci numbers which are of late start were found by focusing on the recursive utilization of two numerical changes used in the prominent four triangle longest-edge bundle, filling in as an outline among computation and numbers. Furthermore a couple of makers spread out a couple of new properties of k-Fibonacci numbers and k Lucas numbers in regards to binomial sums.
Falcon and Court focused on 3-layered k-Fibonacci twistings pondering numerical point of view. A couple of characters for k-Lucas numbers may be found. A couple of makers various properties of k-Fibonacci numbers are gotten by straightforward disputes and related with assumed Pascal triangle.
The place of the ongoing paper is to spread out affiliation recipes between k-Fibonacci and kLucas numbers, thusly deciding a couple of results out of them. In the going with region we investigate a couple of consequences of k-Fibonacci numbers and k-Lucas numbers. Anyway the results can

[7] Bolat, A. Ipeck, and H. Kose, "On the sequence related to Lucas numbers and its properties," Mathematical Aeterna, vol. 2, no. 1, pp. 63-75, 2012.

be spread out by acknowledgment method as well, Binet's condition is overwhelmingly used to exhibit all of them. For different values of m, we have various results:
If m = 0 then [Fk,n] [Lk,2n] = [Fk,3n] - [(-1)n] [Fk,n], n [greater than or equal to] 1 If m = 1 then [Fk,n] [Lk,2n+1] = [Fk,3n+1] - [(-l)n] [Fk,n+1], n [greater than or equal to] 1 and so on.

CONCLUSION

Since various properties, uses of Fibonacci numbers and those of its theories are known, these relations are crucial. Using these relations, properties and usages of Fibonacci numbers and its hypotheses can be moved to Lucas numbers and its theories.

Fibonacci numbers have magnificent and amazing properties; but some are essential and known, others find wide degree in research work. Fibonacci and Lucas numbers cover a broad assortment of interest in present day math.

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