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No Idle Constraint In Flow Shop Scheduling With Separated Set-Up Times: Minimization Of Rental Cost Of Machines

Shakuntla Singla, *Associate Professor,Department Of Mathematics,M.M.E.C., (Deemed To Be University)* Mullana, Ambala, Haryana, India. Shakus25@Gmail.Com
Jatinder Kaur, Associate *Professor And Head Of Department Mathematics, Guru Nanak Girls College,*Yamunanagar, Haryana, India.Jatinderkaur.Gng@Gmail.Com
Harshleen Kaur, *Research Scholar, Department Of Mathematicsm.M..E.C., (Deemed To Be University)*Mullana, Ambala, Haryana, India Harshleenkaur34953@Gmail.Com
Deepak Gupta, *Professor And Head Of Department Mathematics,M.M.E.C., (Deemed To Be University)*Mullana, Ambala, Haryana, India

Abstract

In the present paper, a flow shop scheduling model in two stage under no idle constraint has been studied where the time taken by machines to set-up is separately considered from the processing time. The probabilities with the processing times as well as with the set-up times are also taken into account. The study's goal is to introduce a heuristic algorithm that, when implemented, offers the optimal jobs sequence processing with the shortest makespan possible, reducing the machine idle time to zero and lowering the machine rental cost. The effectiveness of the proposed approach is demonstrated through a numerical sample.

Keywords— scheduling, no idle, set-up time, rental cost, flow shop, optimal sequence.

I. INTRODUCTION

The problem of deciding when to perform given jobs with the purpose of optimizing a function while taking attention of chronological constraints and be located in the limitation resources is known as scheduling. The procedure of sharing the same pre described order of all the machines by the jobs is known as Flow Shop Scheduling. The critical constraint in an industrialized flow shop scenario is the no-idle time on machines or the inability to halt a machine after it has been started. As a result, there can be no downtime for the machines as they must run continually. In the past five decades, there has been considerable attention paid to solve the problem of scheduling. However, Johnson[1] prepared the first triumphant mathematical model that successfully acquired an optimal solution for the two and three stage flow shop scheduling problem. The efficacy of Johnson's model garners significant attention from numerous researchers, who are inclined to explore this avenue. The research conducted by Ignall, E., & Schrage[2], Dannenbring D.G. [3], J. R. Jackson[4], Yoshida and Hitomi[5] expanded upon their original work by considering a range of parameters and employing different optimality criteria.

No-idle flow shop scheduling entails no-idle constraints, which means that machines constantly operate with no breaks. The first investigation of the m-machine no-idle condition in a flow shop was conducted by Adiri and Pohoryles[6]. A three-stage flow shop scheduling issue with the target of total flow time under no-idle situations was the subject of another algorithm published by Narain and Bagga[7]. An approach to reduce rental cost for the no idle two-stage flow shop scheduling problem that takes job weighting into account was provided by Gupta, Goel & Kaur [8]. A rental cost-minimization technique for no idle two-stage flow shop scheduling that takes weightage & transit time into consideration was provided by Gupta et al.[9]. Nature is an ocean of knowledge that motivates living creatures to discover answers to their intricate problems. Additionally, researchers applied this knowledge to solve complex engineering challenges. Several noteworthy references relevant to handle optimization tactics are the works by Malik et al.[10] ,Kumari et al.[11],Singla, Modibbo, Mijinyawa, Malik, Verma& Khurana[12],Sunita et al.[13][14].
Singh T.P.et.al. [6] with the intention to minimize the cost of machines which is consumed

on rent studied Flow shop scheduling model in two stage together with the concept of job-block. Further Gupta D.et.al. [7] widened the study by considering separated set-up times from processing times and both allied with probabilities with the same objective as in [6]. Palmer [11] applied the heuristic approach for minimizing make-span in n-job m- machine problem. Gupta D.et.al. [9],[11] studied Flow Shop Scheduling models in two stage with the idea to optimize the total of the waiting time of all the jobs where the parameters like job block concept, separated set-up times are well thought of. Also, this paper makes an effort to broaden Gupta Deepak[16] research by incorporating the significant jobs in a string of disjoint job block. Identifying the most optimal order to complete jobs in order to save down on expensive machine rentals is the focus of the current study.

II. PRACTICAL SITUATION

Industrialized units play an imperative role in the monetary development of a country. Flow shop scheduling happens in banks, airports, factories etc. Regular working in industries and factories has diverse jobs which are to be practiced on various machines. The idea of lessening the total of the waiting time for all the jobs may be a reasonable aspect from managers of Factory /Industry perspective when he has contract to made the work with less waiting with a viable party to finish the work.

A. *Assumptions*

- Two machines, M_1 and M_2, process the jobs independently of one another in the following order: JK with no allowance of any inter-machine transfer.
- There is no way for two machines to process on the same job at the same time.
- Until a job that is being executed can't be finished, the machines' path of action cannot be altered.
- Calculating utilization time does not take machine breakdown or setup times into account

B. *Rental Policy*

The machines are rented on as needed basis and subsequently return them once they are no longer necessary. Specifically, the initial machine acquired through a rental agreement at the commencement of job processing. Subsequently, the second machine will be obtained on a rental basis once the initial job on the first machine has been completed.

III. NOTATIONS

I :	Jobs sequence 1,2,…, n
s1:	Optimal sequence using Johnson's technique
mi1:	First machine's i-th job processing time
mi2:	Second machine's i-th job processing time
Pi1:	Probability pertaining to mi1
Pi2:	Probability pertaining to mi2
Ti2:	Second machine's i-th job completion time
Wi:	Weightage of i-th job
u1(s1):	Utilization time required for machine M1 in sequence s1
u2(s1):	Utilization time required for machine M2 in sequence s1
c1:	Hiring charges of machine M1 per unit time
c2:	Hiring charges of machine M2 per unit time
l2	Latest time to hire machine M2 to vanish idle time
r(s1):	Rental cost for sequence s1

IV. PROBLEM FORMULATION

The Machine M1 and M2 are dealing out n jobs in the sort M1M2, m1j and m2j are the processing times of the j-th with probabilities p1j and p2j , on machines M1 and M2 correspondingly. sij and s2j are the set up times with probabilities q1j and q2j of machines M1 and M2 correspondingly after processing j-th job such that $\sum n\ j=1\ pij = \sum n\ j=1\ qij$; i=1,2. The formulation of the problem in matrix form as defined by Gupta D.et.al.[7] can be seen in Tab.1. Our goal is to come across a best possible sequence Sj of jobs.

TABLE I. PROBLEM FORMULATION IN MATRIX FORM

JOB	Machine A		Machine B	
α	p_α	S_α^A	q_α	S_α^B
1	p_1	S_1^A	q_1	S_1^B
2	p_2	S_2^A	q_2	S_2^B
3	p_3	S_3^A	q_3	S_3^B
-	-		-	
-	-		-	
n	p_n	S_n^A	q_n	S_n^B

V. HEURISTIC ALGORITHM

Step1: Calculate expected processing time A'_α and B'_α on machine A and machine B respectively as follows.

(i) $A'_\alpha = p_\alpha - S_\alpha^B$

(ii) $B'_\alpha = q_\alpha - S_\alpha^A$

Step 2: Apply Johnson's method to obtain sequence S' which minimizes the total elapsed time.

Step 3: Prepare the In-Out table for the sequence S' obtained in step2 and obtain the total elapsed time $t_{\alpha,2}$

Step 4: Compute

$$K_2 = t_{\alpha,2} - \sum_{\alpha=1}^{n} B'_\alpha$$

Step 5: Take the latest time K_2 to start processing on machine B.

Step 6: Prepare In –Out table for the machines with K_2 as the latest time for machine B

Step 7: Calculate utilization time $U_1(S)$ and $U_2(S)$ of machines A and B by

$$U_1(S) = \sum_{\alpha=1}^{n} A'_\alpha$$

$$U_2(S) = t_{\alpha,2} - K_2.$$

Step 8: Finally calculate

$$R(S') = U_1(S) * C_1 + U_2(S) * C_2$$

VI. NUMERICAL ILLUSTARTION

Assuming two machines M1 and M2 are processing 5 jobs in Flow Shop in Tab. 2. The hiring cost for per unit time for machine A and machine B are 4units and 6 units respectively.

TABLE II. PROBLEM FORMULATION IN MATRIX FORM

Jobs	Machine A		Machine B	
A	p_α	S_α^A	q_α	S_α^B
1	4	2	6	3
2	6	3	4	2
3	5	1	3	3
4	3	2	5	2
5	8	2	2	2

Solution

Step1: Define new expected processing times A'_α & B'_α on machines A & B respectively as shown in the table 5.3

TABLE 5.3: Expected processing time of jobs

Jobs	A'_α	B'_α
1	4-3+1=2	6-2+1=5
2	6-2+2=6	4-3+2=3
3	5-3+1=3	3-1+1=3
4	3-2+3=4	5-2+3=6
5	8-2+2=8	2-2+2=2

Step 2: Obtain the Johnson's sequence S'=1, 4, 3, 2, 5.

Step 3: For the optimal sequence S', prepare In – out table as in table 5.4

TABLE 5.4: Flow table for optimal sequence

Jobs	In-out	In – out
1	0-2	2-7
4	2-6	7-13
3	6-9	13-16
2	9-15	16-19
5	15-23	23-25

As per step 4: Calculate $K_2 = 25 - (5+3+3+6+2) = 6$

As per step 5: Arrange the In- out Table with K_2 as starting time for machine B, we observe in table 5.5, the idle time will be zero.

TABLE 5.5: In - out table with K_2 as starting time for machine B

Jobs	In – out	In – out
1	0-2	6-11
4	2-6	11-17
3	6-9	17-20
2	9-15	20-23
5	15-23	23-25

Step 6: $R(S') = \sum_{\alpha=1}^{n} A'_\alpha * C_1 + U_2(S) * C_2$

$= 23*4+19*6$

$= 92+114=206$ units.

Hence the above calculated results obtained for machine route $M_1 \rightarrow M_2$ of the optimal sequence $s_1=\{1, 2, 4, 5, 3\}$ are described in TABLE III.

TABLE III. COMPARATIVE ANALSIS OF RESULTS

Machine Route $M_1 \rightarrow M_2$	Utilization Time of M_2	Rental Costs
Proposed Algorithm	17.2 units	172.8 units
Johnson Algorithm	20.4 units	192.0 units

Hence from the above TABLE III. , we conclude that the proposed heuristic algorithm created for machine route $M_1 \rightarrow M_2$ provides the minimum utilization time and rental cost for optimum solution s_1.

VII. COMPUTATIONAL EXPERIMENTS

To check the effectiveness of the proposed method, a number of several examples of various groups are randomly considered in which each group upon different number of jobs. Here seven groups are generated with job sizes 5, 10, 20, 30, 50, 55, 70 and each group is observed over 10 different arbitrarily generated tribulations. The job 4 and job 2 has been considered as a block in all groups. The mean of the total waiting time of each problem for proposed algorithm is compared with the mean of already existed make-span approaches of Johnson [1] and Palmer [11] shown in Table 6 and are plotted in graph as shown in Fig.1, which reveals that the curve of proposed method is lower than the other two curves whereas Palmer's algorithm curve is high among all.

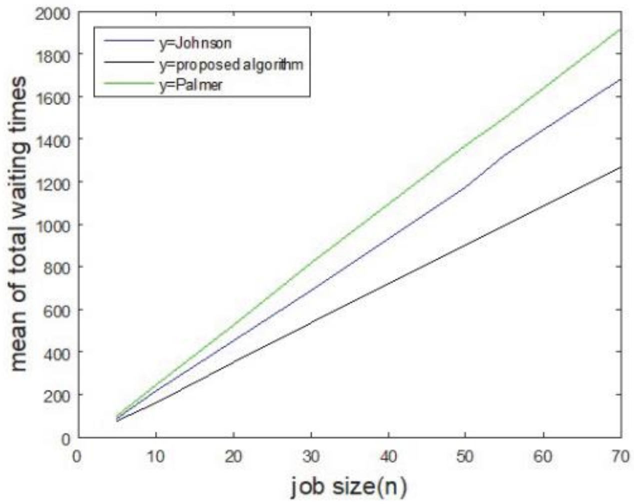
TABLE 6. Comparison of Computational results

No.of Jobs	Mean Waiting Time of Jobs (Johnson's Method)	Mean waiting Time of Jobs (Palmer's method)	Mean waiting Times of Jobs (Proposed method)
5	89.66	99.44	77.89
10	215.86	246.37	161.37
20	454.27	523.03	350.27
30	690.97	815.68	539.30
50	1173.70	1366.33	900.41
55	1323.54	1495.07	991.60
70	1681.69	1916.85	1265.14

In addition, the percentage of error for each of the problem is also calculated by using the formula

$$err = [(W\delta - W\theta)/W\theta] * 100,$$

where $W\delta$ is the total waiting time of existed algorithms and $W\theta$ is the total waiting time of the same job computed by using proposed algorithm. For the sake of measuring the wellness of the proposed algorithm, mean of percentage error is calculated for all job groups and then figured out in the graph below ,shown in Fig. 2. Furthermore it can be seen that Palmer's algorithm produces an error significantly larger than the Johnson's algorithm.



Comparison of Computational results

TABLE 7. Mean of percentage errors

N	Mean of percentage error of total waiting times in Johnson's Algorithm	Mean of percentage error of total waiting times in Palmer's Algorithm
5	15.48	27.98
10	34.26	53.62
20	29.82	49.52
30	28.29	51.51
50	30.37	51.85
55	33.53	50.82
70	33.02	51.68

VIII. CONCLUSION

The present paper deals with No Idle Constraint in Flow Shop scheduling Models with separated set up times incorporating a group of two jobs as a block and proposed heuristic method which provides a near optimal schedule to minimize the total waiting time of jobs. The computational experiments shows that the

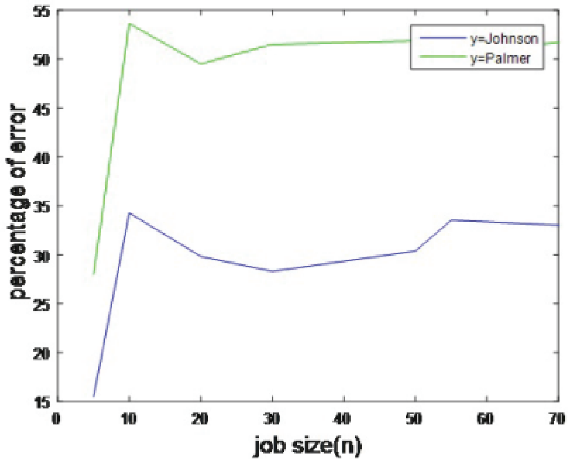


Fig. 1. Mean of percentage errors

TABLE 8. Average of mean percentage errors

No.of Jobs	Average of mean percentage errors
Palmer's	48.14
Johnson's	29.25

approaches of Johnson [1] and Palmer[11] no doubt minimize the completion time but they however delay the jobs to be processed from first machine to second machine. The proposed algorithm keeps in mind not to make jobs too much wait for processing on second machine when they got free from first machine. The objective of minimizing the waiting time of jobs will be significant to manager's point of view when he has contract with the party to complete their job without making too much wait once the process started.

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