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A Mathematical Study of Simplex Method for Linear Optimization

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Abstract

The Simplex Method is one of the most powerful and widely used techniques in Operations Research for solving linear optimization problems. This study presents a comprehensive mathematical exploration of the Simplex Method, focusing on its theoretical foundations, computational procedure, geometric interpretation, and practical significance in decision-making processes. Linear optimization problems arise in diverse fields including manufacturing, logistics, finance, transportation, and resource allocation, where solutions must satisfy system constraints while maximizing or minimizing an objective function. The Simplex Method provides an efficient algorithmic approach to navigating feasible solution spaces to obtain optimal results.

This study begins by examining the mathematical structure of linear programming problems, including objective functions, decision variables, constraints, slack variables, and feasible regions. The geometric interpretation of linear programming is analyzed through convex polyhedrons, where the Simplex Method systematically moves along the vertices of the feasible region toward the optimal vertex. A detailed analysis of the algorithmic steps—forming the initial tableau, identifying pivot elements, performing row operations, and iterating until optimality—is provided to highlight the method's precision and logical progression.

The research further investigates the conditions under which the Simplex Method operates efficiently, including non-degenerate solutions, bounded feasible regions, and feasible starting points. Special cases such as degeneracy, unbounded solutions, infeasible LP problems, and multiple optimal solutions are also discussed, emphasizing how these affect algorithm convergence. Duality theory is introduced to deepen the mathematical understanding of optimality and shadow prices, demonstrating the interconnected relationship between primal and dual linear programming models.

Through computational experiments and example-based analysis, the study evaluates the performance of the Simplex Method in solving both small-scale and large-scale optimization problems. The results confirm that the method consistently finds optimal solutions within a limited number of pivot operations, making it highly effective for real-world applications. Additionally, the advantages of the Simplex Method—such as its structured approach, adaptability to various problem sizes, and ability to incorporate sensitivity analysis—are highlighted.

Overall, this study concludes that the Simplex Method remains one of the most fundamental and reliable algorithms in mathematical optimization. Its strong theoretical framework, combined with practical applicability, continues to influence advanced optimization techniques and modern computational tools. Future research may explore the integration of the Simplex Method with interior-point methods, artificial intelligence–based optimization, and software-driven simulation models to enhance efficiency and real-time decision-making capabilities.