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Subscription Inventive Over the S-Convex Set In Linear Space

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Abstract

In the context of the present paper, our focus revolves around the exploration and analysis of S-convex sets, which represent a fascinating and novel extension of the classical concept of convex sets. We define S-convex sets and discuss their main properties as well as the corresponding S-convex hull. Moreover, we clarify the concept of S-linear combinations, which are essential to our research. This first work lays the foundation for a thorough study of S-convex sets and their characteristics. A number of examples are included to show the necessity of the hypotheses of various theorems.

Keywords: S-Convex Set, Analysis

1. Introduction:

Convexity is a fundamental and widespread concept that dates back to Archimedes (c. 250 B.C.), and his famous measure of the estimation of *P* (using encircled and recorded standard polygons). He observed the important fact that a convex function's parameters are smaller than those of another convex function that encompasses it. We perceive convexity continuously and from multiple perspectives as a matter of certainties. Our upright posture is the least distracting example, and it remains valid as long as our center of gravity's vertical projection is contained within the convex shape of our feet. Furthermore, convexity has a significant impact on our day-to-day lives due to its numerous uses in commerce, industry, medicine, craftsmanship, and other areas. Issues with the optimal distribution of resources and the balance of pleasurable recreations are also relevant. Even though a convex set is a particular kind of linear space, its research is essential to the advancement of linear spaces such as Hilbert space, connected space, normal linear space, branch spaces, and so on. It provides a concise overview of contemporary mathematics. Since a raised capacity is one whose epigraph is a convex set, the theory of convex functions is a subset of the general subject of convexity. Either way, it's a significant hypothesis that touches on almost every aspect of science. The graphical analysis is most likely the main topic that contributes to the significance of the encounter with this theory. The use of Hessian as a higher dimensional conjecture of the second derivatives then arises, along with the problem of determining the very estimates of functions with several variables. The next step is to go on to advancement problems in infinite dimensional spaces; however, despite the specialised attention required to handle such problems, the basic ideas are actually similar to the one variable situation. We recommend Jensen for a thorough analysis. In any case, he wasn't the first to do these kinds of tasks. Over the course of the twentieth century, there was a remarkable research activity in mathematical economics, nonlinear optimisation, convex analysis, and related fields that produced important discoveries. Number of benevolent act have, been carried out of linear spaces. A record of all these may be observed as work of Paul R.Halmos [01], John Harvarth [02] , George Finlay Simmons[03], A.P. Robertson [04], A. Charnbolle [13], R. Finn [14], Schneider [16], W.P. Ziemer [17], and many more. Later on by imposing a set of suitable conditions on the Convex Set, F. Alter, et. al, [09] G Bellettini, et. al, [10,11], G Bellettini, et.al. [12], C. Rosales [15], contributed the concept of S-Convex Set, which is more results using the notion of S-Convex Set which is analogous to that for Convex Sets.

2. Objectives :

scope of doing work with the concept of S convex set. We tend to propose to study similar sets and efforts will be made to establish some of the results with the concept of S- convex set with the study of the convex hull in a real linear space.

3. Results

Use of specific procedure or technique will be used to identify, select, process, and analyze information about the S convex set. We tend to conjointly propose to look at a circled set in a real linear space and shall observe that whether the S convex set it is also becomes a circled set then we will commit to establish some of the outcomes just like to that for the convex set. The purpose of this paper is to discuss under what conditions two disjoint convex subsets of a linear topological space can be separated by a continuous linear functional. Such separations are often described in geometric terms by discussing separation by closed hyper planes. A hyper plane may be defined as a translate of the kernel of a non-trivial linear functional. Equivalently, a hyper plane is a translation of a subspace of dimensions one. In a linear topological space a hyper plane is closed if and only if it is the kernel of a continuous linear functional. This leads to the geometric interpretation that two sets can be separated by a continuous linear functional if and only if it is possible to slip a closed hyper plane between them

3.1 LINEAR SPACE:

The symbol K will stand for either the set R of all real numbers or the set C of all Complex numbers.

A structure of linear space on a Set E is defined by two maps:

- $(x, y) \rightarrow x + y$ of $E \times E$ into E and is called Vector addition.
- $(\alpha, x) \rightarrow \alpha x$ of $K \times E$ into E and is called Scalar multiplication.

The above two maps are assumed to satisfy the following conditions:

- $x + y = y + x$ for every x, y in E.
- $x + (y + z) = (x + y) + z$ for every x, y, z in E.
- There exists an element O in E such that $x + O = O + x = x$ for every x in E.
- For every element x in E there exists an element denoted by x in E such that, $x + (-x) = (-x) + x = O$, for every x in E.
- $a (x + y) = ax + ay$, for every a in K and all x, y in E.
- $(a + b) x = ax + bx$, for every a, b in K and all x in E.
- $1.x = x$ for every x in E.

Whenever all the above axioms are satisfied, then the set E is known as a Linear Space (or Vector Space) over K. Now if K, be the set of all real members, then E is a real linear space and similarly if K be the set of all complex numbers, then E is called a Complex Linear Space. Here, every element of E is called a Vector and every element of K is called a Scalar. The Zero Vector O is unique and called the zero elements or the Origin.

3.2 LINEAR SUBSPACE:

Let E be a linear space over a field K. A non empty subset F of E is called a Linear Subspace (or Simply Subspace) of E, if F itself forms a vector space over K with respect to the addition and scalar multiplication defined on E.

3.3 CONVEX SET:

Let S be a non-empty subset of a linear space E now for $x, y \in S$ and $\lambda, \mu \geq 0$, then S is called a Convex Set whenever,

$$\lambda x + \mu y \in S \text{ for } \lambda + \mu = 1.$$

The below figures represent the difference between convex set and non-convex set.

3.4 S-CONVEX SET:

Let A be a non empty subset of a linear space E. Now, for every $x, y \in A$ and $\lambda, \mu \geq 0$ b (scalars) we shall say that A is a S-Convex Set if, $\lambda x + \mu y \in A$ for $\lambda+\mu \leq 1$. Thus, it is observed that, every S-Convex Set is a Convex Set but the converse may or may not be true. In this way a S-convex set is more general than a Convex Set. Equivalently a Convex Set is a particular case of S-Convex Set.

3.5 LINEAR COMBINATION:

Let E be a linear space over the field of scalars K. We now consider an expression of the form,

$$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \ldots\ldots\ldots\ldots + \lambda_n x_n \qquad \ldots(1)$$

Where, $\lambda_i \geq 0$, for every $i = 1, 2, 3, \ldots, n;$ and n being a positive integral such that $x_i \in E$ for each i. Then the combination (1.1) is called a Linear Combination. We shall denote the set of all linear combination of the set A by $L(A)$.

3.5 S-LINEAR COMBINATION:

Let A be a S-Convex set in a Linear Space E over the field K. Now, consider the expression,

$$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \ldots + \lambda_n x_n \qquad \ldots(2)$$

Where, $0 \leq \lambda_i \leq 1$, for every $i = 1, 2, 3, \ldots, n$; and n being a positive integral such that,

$$\sum_{i=1}^{n} \lambda_i \leq 1,\ \ x_i \in A \text{ for } i = 1,2,3,\ldots,n$$

Then the expression (2) is known as S-Linear Combination of the elements of A. It is also known as S-Convex Combination.

3.6 INTERSECTION OF ALL S-CONVEX SETS CONTAINING A [P(A)] :

Let A be a subset of a linear space E then, $P(A)$ is the intersection of all S-Convex Sets containing A.

SUITABLE MAPPING:

Let E and F be two Linear Spaces defined over the same scalar field K. Let f be a mapping from E to F, which satisfied the following two conditions:

(i) $f(u+v) = f(u) + f(v)$, for every $u, v \in E;$

(ii) $f(\lambda u)=\lambda f(u)$, for every $u \in E, \lambda \in K$ ($K \cong R$, the set of all real numbers).

Here, we have the liberty to express conditions (i) and (ii) together as,

$$f(\lambda u + \mu v) = \lambda f(u) + \mu f(v)$$

for every $u, v \in E$ and $\lambda, \mu \in K$ ($K \cong R$, the set of all real numbers.)

It may be written as f_u in place of $f(u)$ for every $u \in E$.

As the structure of the mapping is linear structure, then this suitable defined mapping is also known as linear mapping.

Also, $f(0) = f(0.0) = 0f(0) = 0$.

Also, $f(-u) = f(u(-1)) = (-1) f(u) = - f(u)$ [or $- f_u$]; for every $u \in E$.

This indicates that mapping f form E into F preserves the properties of origin and negatives.

ZERO LINEAR MAPPING:

Let E and F be two linear spaces over the same scalar field K ($K \cong R$, the set of all real numbers).

Let $f: E \rightarrow F$ be a mapping from E into F defined by,

$f(u) = 0$ (Zero vector of F) for every $u \in E$

Then, Let $u, v \in E; \alpha, \beta \in K$ and $\alpha u + \beta v \in E$

Also, $f(\alpha\mu+\beta v) + 0$ {by definition of f]

$= 0+0$

$= \alpha.0 + \beta.0$

$= \alpha f(u) + \beta f(v)$

That is, $f(\alpha u + \beta v) = \alpha f(u) + \beta f(v)$

Thus, it is observed that this Zero Mapping f defined in such a way that it is a Linear Mapping. Hence, it is known as Zero Linear Mapping. Propose to study the role of a convex set in a topological vector space X and endeavors are going to be created to indicate outcomes that if "A" is a S- convex set in "X" then "A" is also a S- convex set in "X". We shall also see whether or not this result stands helpful for Union and intersection of any two to S-convex sets too.

4. Conclusion:

Propose to introduce the concept of the balanced extension of a set "A" in terms of S- convex set and endeavors will be made to establish some of the results utilizing the concept of S-convex set. On account of the study projected it is trustworthy that the work will get a end result, if not we have a tendency to create associate in allied topics which can emerge through the literature survey. In a linear topological space, interior points are internal points. Also, linear functional that separate convex sets turn out to be continuous, provided one of the convex sets has non-empty interior. Thus any two disjoint convex sets may be separated by a closed hyper plane provided one of the sets has non-empty interior.

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