Study on a Nonlinear Programming Containing Support Functions

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Abstract: In mathematical programming, there are various research articles that have talked about duality theory widely for a program containing the square root of a positive semi definite quadratic capacity by a few creators, e.g., Chandra et.al], Zhang and Mend and the references referred to there. The prevalence of this sort of problems lies in the way that, despite the fact that the objective and constraint functions are non-differentiable, a basic formulation of the dual might be given. Non smooth mathematical programming theory manages significantly more broad sort of functions that require summed up sub differentials] or semi differentials. Be that as it may, the square root of a positive semi unequivocal quadratic frame is one of few instances of a non-differentiable capacity whose sub differential or semi differential can be composed unequivocally. Here a term with square root of a positive semi unequivocal quadratic frame is supplanted by a to some degree more broad capacity, in particular, the help capacity of a minimized raised set, the sub differential can be basically communicated. These observations have inspired us for this research displayed in this section.

Keywords: Mathematical programming, Karush-Kuhn-Tucker optimality, Notations

1.1 Introduction

In this study, , a blended sort dual to a programming issue containing bolster functions in objective and additionally constraint functions, is defined and different duality results are approved under summed up convexity and invexity conditions. A few known results are found as extraordinary cases. In area 2.2 optimality conditions are inferred for a nonlinear fractional program in which a help work shows up in the numerator what's more, denominator of the objective capacity and in addition in every constraint work. As a utilization of these optimality conditions, a dual to this program is defined what's more, different duality results are set up under summed up convexity. A few referred to results are reasoned as extraordinary cases. For this nonlinear fractional program, second request dual is detailed and different duality results are built up under fitting summed up convexity and summed up invexity. In this segment, exceptional cases are likewise created.

2.1 Mixed Type Duality for a Programming Problem Containing Support Functions.

2.1.1 Introductory Remarks

In, Husain et.al. Considered the following non differentiable nonlinear problem with support functions:

(NP): Minimize f(x) + s(x| C)

Subject to

$$g^j(x) + s(x|D^j) \leqq 0, \; j = 1, 2, \ldots, m.$$

Where,

(I) For the n-dimensional Euclidean space R^n, f: $R^n \rightarrow R$ and g_j : R^n – (j = 1,2,..., m), are constantly differentiable, and

(ii) s (.| C) and s (.| D_j), (j = 1, 2, ..., m) are individually the help functions of raised minimized sets C and D_j, (j = 1, 2 , m) in R^n.

(iii) Husain et. al.[62] set up the accompanying Fritz John sort fundamental optimality conditions for the issue (NP).

Proposition 2.1 (Fritz John type optimality conditions) : If $\bar{x}$ is an optimal solution of (P), there exist Lagrange multipliers $\bar{\gamma}$ ∈R .γ ∈ R^m with $\bar{\gamma}$ = (y.... $\bar{\gamma}_m$)^,$\bar{z}$ ∈ R^n and $\bar{w}$ ∈R^n,(j = 1,2 m) such that

$$\bar{\gamma}\nabla\left(f(\bar{x})+\bar{x}^T\bar{z}\right)+\sum_{j=1}^{m}\bar{y}_j\nabla\left(g_j(\bar{x})+\bar{x}^T\bar{w}_j\right)=0,$$

$$\sum_{j=1}^{m}\bar{y}_j\left(g_j(\bar{x})+\bar{x}^T\bar{w}_j\right)=0,$$

$$\bar{x}^T\bar{z}=s(\bar{x}\,|\,C),$$

$$\bar{x}^T\bar{w}_j=s(\bar{x}\,|\,D_j),\ j=1,2,\dots,m,$$

$$z\in C,\ \bar{w}_j\in D_j,\ j=1,2,\dots,m,$$

$$(\bar{\gamma},\bar{y})\geq 0,$$

$$(\bar{\gamma},\bar{y})\neq 0.$$

At the point when Slater's constraint capability is fulfilled at x, the above Fritz John optimality conditions move toward becoming Karush - Kuhn - Tucker optimality conditions, as this state's inspiration of multiplier y related with the objective capacity. Utilizing these optimality conditions, the accompanying Wolfe sort dual to the issue (NP) was detailed in [62] and the duality results were inferred under convexity of f and g.

(WD) : $\quad \text{Maximize } f(u)+u^Tz+\sum_{j=1}^{m}y_j\left(g_j(u)+u^Tw_j\right)$

Subject to

$$\nabla\left(f(u)+u^Tz\right)+\sum_{j=1}^{m}y_j\nabla\left(g_j(u)+u^Tw_j\right)=0,$$

$$z\in C,\ w_j\in D_j,\ j=1,2,\dots,m,$$

$$y\geq 0.$$

Further the authors in [62] debilitate the convexity necessities in Wolfe duality by building the accompanying Mond-Weir sort dual:

(M–WD) : $\quad \text{Maximize } f(u)+u^Tz$

Subject to

$$\nabla\left(f(u)+u^Tz\right)+\sum_{i=1}^{m}y_i\nabla\left(g_i(u)+u^Tw_i\right)=0,$$

$$\sum_{j=1}^{m}y_j\nabla\left(g_j(u)+u^Tw_j\right)\geq 0,$$

$$z\in C,\ w_j\in D_j,\ j=1,2,\dots,m,$$

$$y\geq 0.$$

and established duality theorems under the hypotheses that $f(.)+(,)^Tz$ is pseudoconvex for all $z\in R^n$ and that $\sum_{j=1}^{m}Y_j\,(q_j(.)+(.)^T w_j)$ is quasiconvex for all $w_j\in R^n$, $j=1,2,\dots m$.

In this area, we propose, in the spirit of Bector, Chandra and Abha what's more, Xu , a blended sort dual to (NP) and build up different duality theorems under summed up convexity and summed up invexity conditions. Extraordinary cases are examined to demonstrate that our results expand some prior results in the literature.

Before introducing a blended sort dual model for (NP), in the following area, we quickly say a few preliminaries for simple reference.

For a set K, the typical cone, to K at a point $x\in K$ is characterized by $N_k(x)=\{y|y^T(z-x)\leq 0,$ for all $z\in K\}$ When K is a minimal raised set, y is in $N_k(x)$ if and just if $s(y|K)=x^Ty$, i.e., x is a sub differential of s at y.

The accompanying Fritz John optimality conditions for a differentiable programming issue are from [96]. Let F: $R^n \to R$, G: $R^n \to R^m$ and H: $R^n \to R^p$ be differentiable capacity and Q be a curved set in R". Consider the accompanying issue:

$$\text{Maximize: } F(z)$$

Subject to

$$G(z) \leq 0,$$

$$H(\xi) = 0,$$

$$\xi \in Q.$$

If $\bar{\xi}$ is an optimal solution to this problem, then there exist a Lagrange multipliers $\tau \in R_+$, $\lambda \in R^m$ and $\mu \in R^p$, not all zero, and q in $N_q(\bar{\xi})$ such that

$$-\tau \nabla F(\xi) + \lambda^T \nabla G(\xi) + \mu^T \nabla H(\xi) + q = 0, \ \lambda^T \nabla G(\xi) = 0,$$

Where R^m denotes nonnegative orthant of R^m

2.1.2 Mixed Type Duality

We formulate the following mixed type dual (Mix D) to (NP):

$$(\text{Mix D}): \ \text{Maximize } f(u) + u^T z + \sum_{j \in J_0} y_j \left(g_j(u) + u^T w_j\right)$$

Subject to

$$\nabla\left(f(u) + u^T z\right) + \sum_{j=1}^{m} y_j \left(g_j(u) + u^T w_j\right) = 0 \qquad (2.2)$$

$$\sum_{j \in J_\alpha} y_j \left(g_j(u) + u^T w_j\right) \geq 0, \ \alpha = 1, \dots, r. \qquad (2.3)$$

$$z \in C, \ w_j \in D_j, \ j = 1, 2, \dots, m, \qquad (2.4)$$

$$y \geq 0, \qquad (2.5)$$

where $J_\alpha \subseteq M = \{1, 2, \dots, m\}$, $\alpha = 1, 2, \dots, r$ with $\bigcup_{\alpha=0} J_\alpha = M$ and

$J_\alpha \cap J_\beta = \phi$, if $\alpha \neq \beta$.

Theorem 2.1 (weak duality): Let x be feasible for (NP) and (u, z, y, w_1 , , w_m) feasible for (Mix D). If for all feasible (u, z, y, w_1, , w_m) ,

$$f(.) + (.)^T z + \sum_{j \in J_0} y_j \left(g_j(.) + (.)^T w_j\right)$$

is pseudoinvex for all z ∈ C and $w_j \in D_j$, j ∈ J_0 and

$$\sum_{j \in J_\alpha} y_j \left(g_j(.) + (.)^T w_j\right)$$

is quasiinvex for all $w_j \in D_j$, j ∈ $J\alpha$, α = 1, 2 , r with respect to the same t|, then

$$\text{infimum (NP)} \geq \text{supremum (Mix D)}.$$

Proof: Since x is feasible for (NP) and (u, z, y, w_1 , , w_m) is feasible for (Mix D), we have, in view of $x^T w_j < s(x|D_j)$, and $w_j \in D_j$ j = 1, 2,…,m, for α = 1 , 2 , r,

$$\sum_{j \in J_\alpha} y_j \left(g_j(x) + x^T w_j\right) \leq \sum_{j \in J_\alpha} y_j \left(g_j(x) + s(x|D_j)\right) \leq 0 \leq \sum_{j \in J_\alpha} y_j \left(g_j(u) + u^T w_j\right)$$

By the quasiinvexity of $\sum_{j \in J_\alpha} y_j \left(g_j(.) + (.)^T w_j\right)$ for all $w_j \in D_j$, $j \in J_\alpha$,

$\alpha = 1, \ldots, r$, this yields, $\eta(x,u)^T \sum_{j \in J_\alpha} y_j \nabla\left(g_j(u) + u^T w_j\right) \leq 0,\ j = 1, \ldots, r$.

Hence,

$$\eta(x,u)^T \sum_{j \in M-J_0} y_j \nabla\left(g_j(u) + u^T w_j\right) \leq 0.$$

From (2.2), it implies that

$$\eta(x,u)^T \left[\nabla\left(f(u) + u^T z\right) + \sum_{j \in J_0} y_j \nabla\left(g_j(u) + u^T w_j\right)\right] \geq 0.$$

$w_j \in D_j$, $j \in J_0$, implies,

$$f(x) + x^T z + \sum_{j \in J_0} y_j\left(g_j(x) + x^T w_j\right) \geq f(u) + u^T w_j + \sum_{j \in J_0} y_j\left(g_j(u) + u^T w_j\right).$$

Since $w_j \in D_j$ and $x^T w_j \leq s(x|\, D_j)$, $j \in J_0$, the above inequality gives

$$f(x) + x^T z + \sum_{j \in J_0} y_j\left(g_j(x) + x^T w_j\right) \geq f(u) + u^T z + \sum_{j \in J_0} y_j\left(g_j(u) + u^T w_j\right).$$

This, form $y_j \geq 0$ and $g_j(x) + s(x|\, D_j) \leq 0$, $j = 1, 2, \ldots, m$, implies

$$f(x) + x^T z \geq f(u) + u^T z + \sum_{j \in J_0} y_j\left(g_j(u) + u^T w_j\right).$$

Because of $x^T z \leq s(x|\, C)$ for $z \in C$, this yields,

$$f(x) + s(x\,|\,C) \geq f(u) + u^T z + \sum_{j \in J_0} y_j\left(g_j(u) + u^T w_j\right)$$

That is,

infimum (NP) $\geq$ supremum (Mix D).

Corollary 2.1: Let $\bar{x}$be feasible for (NP) and ($\bar{u}$, $\bar{z}$, $\bar{y}$, $\bar{w}$,....$\bar{w}_m$)be feasible for (WD) with corresponding objective values be equal. Let the hypotheses of Theorem 2.1.hold. Then $\bar{x}$ is optimal for (NP) and ($\bar{u}$, $\bar{z}$, $\bar{y}$, $\bar{w}$,....$\bar{w}_m$) is optimal for (Mix D).

Theorem 2.2 (Strong Duality): If $\bar{x}$ is an optimal solution of (NP) and Slater's constraint qualification is satisfied at $\bar{x}$, then there exist $\bar{y} \in R^m$, with $\bar{y} = (\bar{y},......\bar{y}_m)$, $\bar{z} \in C$ and $w_j \in D_j$, j = 1 ,2 ...... m such that ($\bar{u}$, $\bar{z}$, $\bar{y}$, $\bar{w}$,....$\bar{w}_m$) is feasible for and the corresponding objective values of (NP) and (Mix D) are equal.

1.5 Conclusions

These problems are not revealed in the literature, but rather if each Ej, (j= 1,2, ... m) are invalid matrices, at that point they surely lessen to the match of the problems, considered by Zhang and Mond [112]. A positive semi-unequivocal matrix A might be written. The duality for these problems has not been studied explicitly in the literature but one can easily establish it on the lines of the results of this research.

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