Mathematical Analysis of Magnetohydrodynamic Flow Over A Stretching Sheet with Heat and Mass Transfer

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Abstract

This article studies the steady two-dimensional flow of a magnetic fluid over a surface that is moving and stretching. The fluid is treated as incompressible, electrically conductive, and follows Newton's laws. The study also looks at how heat and mass are transferred along with the flow. A uniform magnetic field is applied across the fluid, and this setup helps in understanding how the magnetic force affects the flow. Using mathematical models and numerical methods, the equations that describe the movement of the fluid, heat transfer, and mass movement are simplified through similarity transformations. These equations are turned into a group of linked nonlinear equations that are easier to solve. To get solutions, the shooting method is combined with a fourth-order Runge-Kutta technique. The results show how three main factors affect the speed, temperature, and concentration of the fluid: the strength of the magnetic field, the Prandtl number, and the Schmidt number. The magnetic field strongly slows down the fluid because of the Lorentz force. The Prandtl and Schmidt numbers increase the thickness of the layers where heat and mass are transferred, respectively. This study is useful for engineers working with electrically conducting fluids because it shows how movement, heat, and mass transfer are connected in magnetohydrodynamic flows.

Keywords: Magnetohydrodynamic flow; Stretching sheet; Heat transfer; Mass transfer; Boundary layer; Numerical analysis

Introduction

The magnetohydrodynamic (MHD) flow has become a significant field of study because of the wider range of application in the field of engineering, applied science, and industrial advanced technologies. MHD is concerned with the behavior of electrically conducting fluids, including molten metals, electrolytes, plasmas and ionized gases, under the influence of magnetic fields. The forces created between the moving fluid and the magnetic field applied by it are electromagnetic in nature and have a great impact on the movement properties of the flow. These interactions are essential in numerous applications in practice, such as the extrusion of polymer, continuous casting of metals, cooling of nuclear reactors, crystal growth during semiconductor production and electromagnetic flow control systems. The fluid motion, temperature profiles and concentration of species in such applications must be tightly controlled to give the product high quality, safe operation as well as efficiency of the process.

Flow over a extending sheet is a classical boundary layer problem that has been given particular attention to because of the applicability to manufacturing and material processing industries. Extrusion of plastic sheets, rubber sheets, hot rolling, glass fibers, wire drawing, and metal spinning are some processes that involve the use of stretching surfaces. The conceptual innovation of a stretching sheet was made to simulate the boundary layer behaviour caused by steady extension of a surface which greatly changes the structure of the fluid flow relative to the cases of stationary or uniformly moving plates. The analysis can be more realistic when the effects of heat and mass transfer are included in the problem because in the vast majority of industrial processes, momentum, thermal energy, and chemical species interact with each other simultaneously. The interplay of these transport mechanisms results in complicated nonlinear and coupled mathematical models, which require powerful analytical/numerical techniques to find solutions.

Adding a magnetic field adds another degree of complication to the flow dynamics. MHD streams Changing the velocity field is possible because the application of a magnetic field produces a Lorentz force that acts counter to the direction of fluid motion. The magnetic damping effect has a large and forceful impact on the hydrodynamic, thermal, and concentration boundary layer thicknesses. Therefore, the magnetic field can be a helpful tool for controlling flow behavior, heat transfer rate, or even instability in electrically conductive fluids. Typical methods of controlling flow may not be practical or sufficient in high-speed, high-temperature industrial processes due to the interaction of magnetic forces in timing with mass and heat transfer processes.

Over the past few decades, numerous researchers have examined multi-scale hydrodynamic (MHD) boundary layer flow over stretching sheets, taking into account various physical processes such as heat radiation, chemical reactions, suction and injection, porous media, non-Newtonian fluid behavior, etc. These studies have significantly advanced our understanding of the phenomena of individual and group travel in a variety of operational contexts. Nevertheless, the combined effects of magnetic fields, heat conduction, and mass dispersion over a long surface remain an active area of study, even with these advancements. Even when taking into account practical boundary conditions and industry-relevant parameters, the nonlinearity of the governing equations and the strong coupling between them and momentum, energy, and species transport continue to be a computational and mathematical hurdle.

This study will create a strong mathematical model for the steady two-dimensional MHD boundary layer flow over a stretching sheet, including the effects of heat and mass transfer, as discussed earlier. The basic equations are developed from fundamental laws of conservation and then converted into a dimensionless form using suitable similarity variables. After that, numerical techniques are used to solve the nonlinear system of equations. The research will then look at how different important parameters affect the velocity, temperature, and concentration distributions. We believe this work will help understand the physical behavior of MHD transport and offer helpful ideas for improving the design and optimization of engineering systems that use electrically conducting fluids.

Mathematical Formulation

The study looks at a two-dimensional, smooth, steady flow of a fluid that can conduct electricity and doesn't change density. The fluid flows over a surface that is stretching. The surface is at y=0, and it stretches along the x-axis. The direction going up from the surface is the y-direction. The fluid is present in the area where y is greater than zero. The speed of the stretching surface is given by Uw(x) = ax, where 'a' is a positive number that shows how fast the surface is stretching, and the stretching happens in the x-direction. Many industrial processes, like metal shaping and making plastic, use such a type of flow.

A steady magnetic field B0 is applied perpendicular to the stretching surface.

Since the fluid can carry electricity, applying this magnetic field creates a force called the Lorentz force. This force acts in a direction that's at right angles to the fluid's movement. The magnetic field slows down the fluid flow and changes how momentum moves in the boundary layer. The study assumes that many real-world and lab-scale MHD flows have low Reynolds numbers. Because of this, the effect of the magnetic field created by the moving fluid is ignored, using a low magnetic Reynolds number assumption.

It is also assumed that the fluid's density stays the same, the flow is smooth, and the fluid's properties like viscosity, how well it conducts heat, and how it spreads mass are constant.

To simplify things, the study ignores effects like heat from friction and heating from electric currents, and only considers the magnetic force acting on the fluid. Using the idea of boundary layer theory, the study uses equations that describe the conservation of mass, movement, heat, and concentration of substances. These equations help understand how mass, heat, and movement are connected in a magnetic field, and they are used in a step-by-step way to solve numerical problems based on similarity.

Assumptions

- The flow is steady and laminar
- The fluid is incompressible and Newtonian
- Induced magnetic field is negligible (low magnetic Reynolds number)
- Viscous dissipation is neglected
- Heat and mass transfer occur simultaneously

Governing Equations

Assuming the flow is steady, two-dimensional, incompressible, and laminar, and it involves magnetohydrodynamic effects over a stretching sheet, the basic laws that govern mass, momentum, energy, and concentration of species are developed by analyzing how the fluid behaves in the boundary layer. Since the velocity in different directions is connected, the continuity equation takes care of mass conservation. A magnetic field that acts perpendicular to the flow is used to control the movement of an electrically conducting fluid. The momentum equation explains how different forces—like inertia, viscosity, and the electromagnetic Lorentz force—balance each other out. As a result of fluid motion and conductive heat diffusion over the boundary layer, the equation for energy balance governs the flow's thermal energy transfer, which includes convective heat transfer. Also, it's important to remember that the species concentration equation regulates mass transfer by adding convective transport and molecule-based diffusion of chemical species. Beginning with these two linked boundary layer nonlinear equations, we can analytically characterize the flow regime, heat transfer, and mass diffusion of the MHD fluid over the stretched sheet. From here, we can proceed with similarity transformations and numerical analyses.

Continuity equation

There is no net change in mass as a result of the fluid's motion, according to the continuity equation, which is also known as the law of conservation of mass. The net rate of mass flow into any fluid element should be equal to the flow out of mass; this is defined by the dependence between the components of velocity in the streamwise (x) and transverse (y) directions when the continuity equation is applied to the steady, two-dimensional, incompressible flow. Thus, the density of the fluid would be maintained constant across the flow field, as any change in velocity in one direction would be balanced out by a corresponding change in velocity in the other direction.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

In the equations associated with the flow of the boundary layer on a stretching sheet, the continuity equation is an essential step in interrelating the velocity components and allows the introduction of a stream function which, by implication, allows the conservation of the mass to be taken into account, and the mathematical formulation of the problem to be simplified.

Momentum equation

The momentum equation governs the change in velocity due to the combined effects of inertial force, viscous force, and electromagnetic force, and it describes the law of preservation of the linear momentum of the fluid flow in the boundary layer. The momentum velocities perpendicular and parallel to the stretching sheet caused by its motion in the flow field of acceleration are represented by the convective terms in this equation.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u$$

The concept of viscous diffusion takes into account the flow resistance caused by the fluid's viscosity, which acts to level velocity gradients and is most prominent at the boundary layer. An additional electromagnetic force acting on the body, known as the Lorentz force, is produced when an externally applied transverse magnetic field interacts with the induced electric current. The flow's velocity drops and the momentum barrier layer thickens because this force acts counter to the direction of fluid motion. A huge component of studying the behavior of magnetohydrodynamic flows across a stretching sheet is the momentum equation, which is the equilibrium of inertia, viscosity, and magnetic damping.

Energy equation

The temperature distribution in the boundary layer of a stretched material sheet is determined by the conservation of thermal energy in fluid flow, which is also known as the equation of energy. The effects of advection on the thermal field are represented by the terms in the equation that describe convection; these terms explain that heat is conveyed by the motion of the fluid both perpendicular to and parallel to the sheet.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

The thermal diffusivity of the material determines the conductive term, which is the heat diffusion due to changes in fluid temperature. It is crucial to include this term when determining the thermal boundary layer thickness. Any slowing of fluid motion by magnetic damping changes the convective heat transfer mechanism, hence any modification to the energy equation in magnetohydrodynamic flow is an indirect result of the field's effect on the velocity field. When studying MHD flows over stretching surfaces, the energy equation is crucial for determining the temperature distribution since the balance between convective and conductive heat transfer mechanisms determines the overall flow.

Species concentration equation

The equation for species concentration, which is a type of mass conservation equation, shows how the mass of a chemical substance is kept steady in the fluid flow. It also helps to explain how the concentration of the substance spreads in the layer near the surface of the stretching sheet. The parts of the equation that describe movement, called convective terms, show how the substance moves with the fluid flow both along the direction of the flow and across the surface, meaning how the moving fluid carries the substance both parallel and perpendicular to the surface.

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}$$

The mass diffusivity of the fluid characterizes the concentration gradient, which is expected to spread the species in the higher concentration regions to the lower concentration regions through molecular diffusion, as represented by the diffusive term. When the fluid's velocity drops, the rate of convective mass transfer varies, which means that the concentration field and the magnetic field are interacting with each other in magnetohydrodynamic processes. The concentration boundary layer depth and most mass transport characteristics are defined by the ratio of convection to diffusion. Diffusion process analysis in MHD flow systems involving transport of chemical species, such as in drying processes, pollution dispersion, and catalytic reactions, relies heavily on the species concentration equation.

Similarity Transformation

The similarity transformation is used to turn the main partial differential equations into a set of linked nonlinear ordinary differential equations, making it simpler to solve using numerical methods. By defining appropriate similarity variables, the flow variables depend on fewer spatial coordinates, effectively changing the two-dimensional boundary layer problem into a one-dimensional form.

$$\eta=\sqrt{\frac{a}{\nu}}y,\psi=\sqrt{a\nu}xf(\eta)$$

where $\psi$is the stream function defined by:

$$u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x}$$

The dimensionless temperature and concentration are defined as:

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

Substitution leads to the following nonlinear ODEs

The system of nonlinear partial equations is turned into a set of linked nonlinear ordinary equations by using similarity variables and dimensionless functions in the boundary layer equations for momentum, energy, and species concentration. These equations describe the dimensionless fields of velocity, temperature, and concentration within the boundary layer and are given by the following values.

The transformed momentum equation governing the flow field is

$$f'''(\eta)+f(\eta)f''(\eta)-[f'(\eta)]^2-Mf'(\eta)=0,$$

in which the dimensionless stream function f(η) and the magnetic parameter Mis, which represents the applied magnetic field strength, are defined.

The energy equation governing heat transfer is expressed as

$$\theta''(\eta) + Pr\, f(\eta)\, \theta'(\eta) = 0,$$

Here, θ(η) represents the dimensionless temperature, and Pr is the Prandtl number, which shows how much more important momentum diffusion is compared to heat diffusion.
The species concentration equation governing mass transfer is given by

$$\phi''(\eta) + Sc\, f(\eta)\, \phi'(\eta) = 0,$$

In this case, ϕ(η) represents the concentration in a non-dimensional form, and Sc is the Schmidt number, which shows the ratio of mass diffusivity to momentum diffusivity. These nonlinear ordinary differential equations, together with their related boundary conditions, form a complete mathematical model that governs the coupled magnetohydrodynamic flow, heat transfer, and mass diffusion over the stretching sheet.The system is highly nonlinear, so numerical methods are needed to find accurate solutions.

Boundary Conditions

The present problem of magnetohydrodynamic flow is developed by defining the boundary conditions with the physical behavior of the fluid at the surface of the stretching sheet and in the free stream region that is distant away on the sheet. No-slip condition is applied at the top of the stretching sheet (η=0) and this demands that the velocity of the fluid at the top of the sheet match the extension speed of the sheet. This is the condition that makes sure that the fluid does not move in a relative manner. Also, since the condition of impermeable sheet is that there is no normal velocity at the sheet, then there is no normal velocity at the surface. Both temperature and concentration are set as constant at the surface with given surface temperature and species concentration that are more than in the ambient. Such conditions would be practical in a situation in which the surface under expansion is kept at constant levels of both thermal and concentration throughout the manufacturing process.
All surface motion has less of an impact on the sheet as its tension increases (η 0), and the sheet starts to behave more like an inert fluid. Based on this, the free stream condition is satisfied because the fluid's velocity is expected to be zero. Similarly, as you move farther from the surface, the fluid's temperature and concentration asymptotically approach its ambient values. The temperature and concentration perturbations induced by the stretched sheet do not go beyond the boundary layer area, thanks to these far-field boundary conditions. When the surface and far-field boundary conditions are combined, it provides a physically consistent foundation for solving the transformed governing equations and an accurate and precise description of the boundary layer's velocity, temperature, and concentration field dynamics.
At the sheet ($\eta = 0$):

$$f(0) = 0, f'(0) = 1, \theta(0) = 1, \phi(0) = 1$$

As $\eta \to \infty$:

$$f'(\infty) \to 0, \theta(\infty) \to 0, \phi(\infty) \to 0$$

Numerical Solution Method

Boundary value problem The nonlinear ordinary differential equations transformed into with boundary conditions form a transformed non linear ordinary differential equation. The problem is transformed into a similar initial value problem by the method of shooting. Unspecified initial conditions, including f 2 (0), 0 -1 (0), 0 -3 (0), and 0 -4 (0), are estimated through an iterative process. The fourth-order RungeKutta method is used to solve the resulting system which is very accurate and stable numerically. When the far-field boundary conditions are met within a specific tolerance a convergence occurs.

6. Results and Discussion

To study how important factors influence the magnetohydrodynamic flow, heat transfer, and mass transfer over a stretching sheet, hydraulic simulations were carried out. To examine how the magnetic parameter M, Prandtl number, and Schmidt number together affect the velocity, temperature, and concentration profiles in the boundary layer, the transformed nonlinear ordinary differential equations were solved for many different values of these parameters. To better understand their real-world importance and how they affect the flow, some parameters were changed one at a time while keeping the others the same.

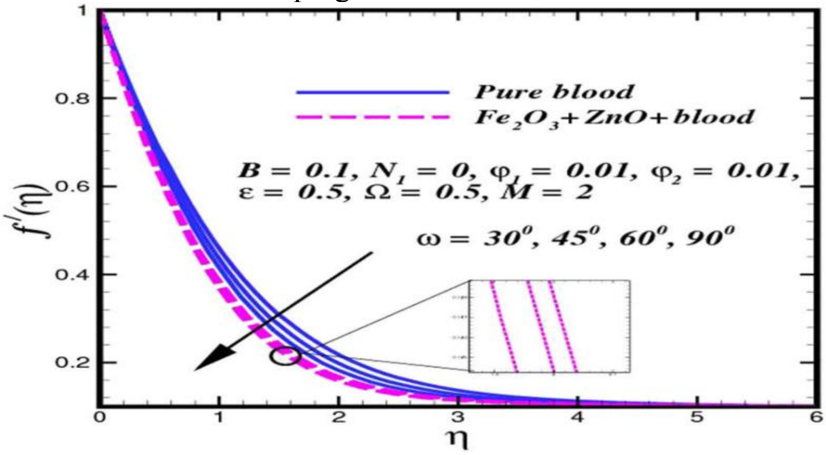

Figure 1: Velocity Profile for Different Magnetic Parameters

The velocity profile shows that an increase in the strength of the magnetic field has a significant effect on decreasing the fluid velocity throughout the boundary layer. The interaction between the supplied magnetic field and the electrically conducting fluid produces the Lorentz force, which is responsible for this phenomenon. By serving as a resistance to the flow direction, the Lorentz force both slows the flow and speeds up momentum diffusion across the stretching surface. This is particularly useful in practice for situations requiring flow stability and control, as it causes the momentumary boundary layer thickness to increase, which in turn increases the fluid's resistance.

The Prandtl number affects how heat moves through a fluid. As seen in the temperature field, when the Prandtl number is higher, the thermal boundary layer becomes thinner. This happens because the Prandtl number shows the relationship between how fast momentum spreads and how fast heat spreads. A higher Prandtl number means heat spreads more slowly compared to momentum. As a result, temperature changes near the surface become steeper, the thermal boundary layer gets narrower, and heat moves more slowly. Liquids like oils and polymer melts behave this way, showing how the properties of a fluid greatly influence how heat is transferred.

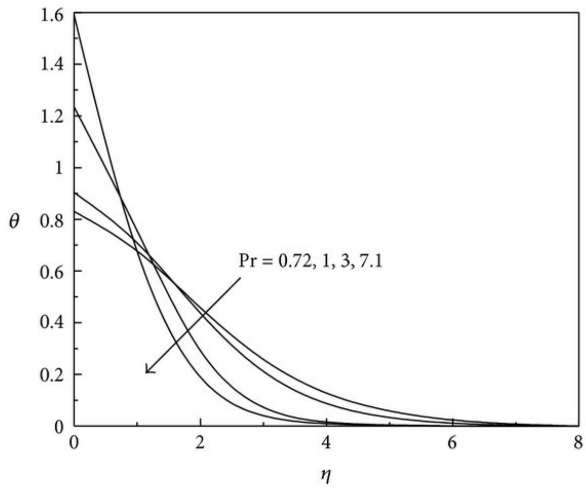

Figure 2: Temperature Profile for Different Prandtl Numbers (Pr)

Similarly, the concentration distribution in the boundary layer can be determined with the help of the Schmidt number Sc. The mass diffusivity of the species is reduced as the concentration of Scleads increases, as the concentration boundary layer thickens to a great extent. Since a lower molecular diffusion velocity is associated with higher Schmidt numbers, concentration gradients are steeper as one approaches the surface. A precise estimate of the rate of mass transfer is crucial in processes such as chemical reactions, drying, and the transportation of contaminants, where this has been determined to be of considerable importance.

In sum, the numerical results indicate that the interchange of momentum, heat, and mass in magnetohydrodynamic fluxes over stretched surfaces is fundamentally connected. By way of governing equations, changes to one transport mechanism have direct nonlinear impacts on the other transport mechanisms. To accurately describe the complex behavior of MHD flow systems, these results show that multi-physical modeling is necessary. The industrial optimization of processes involving electrically conducting fluids and the improvement of thermal and mass transport control can both benefit from the findings of this study.

7. Conclusion

To study the two-dimensional flow of a magnetohydrodynamic fluid over a smooth, expanding surface while also considering heat and mass transfer, this article presents a detailed mathematical model. The complex nonlinear partial differential equations that describe the system were simplified into a set of connected nonlinear ordinary differential equations using similarity transformations. These equations were then solved numerically using the Runge-Kutta shooting method. The study looked at how key factors influence the flow, temperature, and concentration profiles. The findings show that as the magnetic parameter increases, the thickness of the momentum boundary layer also increases, which causes a noticeable decrease in fluid velocity because of the Lorentz force. Higher Prandtl numbers reduce the ability of heat to spread, which makes the thermal boundary layer thinner. Similarly, higher Schmidt numbers reduce mass diffusion, making the concentration boundary layer thinner. The research highlights how momentum, heat, and mass transfer are connected in MHD flows over stretching surfaces. These results have important applications in fields like temperature control systems, polymer processing, and metalworking. Future work could include factors like heat radiation, chemical reactions, energy loss from viscosity, or behavior of non-Newtonian fluids in the current model

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