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Solving Solid Transportation Problem with Carbon Emission Constraints

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Abstract

The Solid Transportation issue (STP) is a three-dimensional version of the traditional transportation issue. It adds a third index, which is the type of conveyance or transportation mode. This makes the model much more useful in real life and gives it many more possible solutions. In this paper, we develop and solve a multi-objective STP with explicit carbon emission constraints, simulating a realistic Indian logistics scenario that includes three supply sources (Ahmedabad, Pune, Hyderabad) and four demand centers (Mumbai, Delhi, Chennai, Kolkata), serviced by four modes of transport (road, rail, air, and sea/inland waterway). The main goal is to keep the overall cost of transportation (₹) as low as possible. The second goal is to keep the total amount of CO_2 emissions below a user-defined limit. We use the North-West Corner Method and Vogel's Approximation Method (VAM) to get an initial Basic Feasible Solution (BFS). Then, we use the Modified Distribution (MODI) method and the Stepping-Stone algorithm to make it better. The findings indicate a 37.7% decrease in carbon emissions with the implementation of a 30-ton CO_2 cap compared to an unconstrained baseline, accompanied by an 11.3% rise in overall transportation costs. Sensitivity analysis shows that moderate emission limitations are the best way to balance economic efficiency and environmental responsibility. The research offers a robust, replicable mathematical paradigm directly relevant to green supply chain decision-making in developing nations.

Keywords: Solid Transportation Problem, Carbon Emission Constraints, Vogel's Approximation Method, MODI Method, Green Logistics, Multi-Objective Optimisation, Linear Programming, Supply Chain Management.

1. Introduction

The backbone of contemporary economies is transportation and logistics. The logistics industry in India processes more than 4.6 billion metric tons of freight per year and accounts for almost 14% of GDP (Ministry of Commerce & Industry, 2023). Indian authorities have made it a priority to incorporate environmental goals into freight optimization models in light of the country's Paris Agreement pledge to cut carbon intensity 45 percent below 2005 levels by 2030. Using supply and demand restrictions, the Classical Transportation Problem (CTP) was formalized by Koopmans (1949) and first proposed by Hitchcock (1941) and seeks to optimize the movement of a single commodity from m sources to n destinations at least cost. By adding k conveyance alternatives as a third dimension, Haley (1962) expanded this model to the Solid Transportation Problem (STP), resulting in a decision matrix with dimensions m × n × k. A shipment from Pune to Delhi via road has significantly different costs, transit times, and emissions compared to the same shipment via rail or air, highlighting the vital importance of dimensional enrichment in practice. Although STP is elegant in theory, the majority of published works on the topic ignore environmental concerns in favour of reducing costs. Organizations must now clearly budget carbon emissions with financial costs due to legal pressure and climate science (e.g., the EU Carbon Border Adjustment Mechanism, India's Carbon Credit Trading Scheme, 2023) and other similar initiatives. To fill that void, this article incorporates a strict CO_2 emission limitation into the STP, making it a constrained optimization problem amenable to both traditional and cutting-edge methods in operations research.

Section 2 provides a literature review of interest. In Section 3, the mathematical formulation is introduced. Section 4 details the empirical evidence. Section 5 details the approach used to find a solution. The computational findings and sensitivity analysis are presented in Section 6. Considerations of both theory and practice are addressed in Section 7. Finally, Section 8 finishes by outlining potential avenues for further study.

1.1 Research Objectives

This study pursues four specific objectives:

1. To formulate a mathematically rigorous STP incorporating carbon emission constraints.
2. To derive an Initial Basic Feasible Solution using VAM and improve it to optimality via MODI.
3. To apply the model to real Indian freight data and quantify the cost–emission trade-off.
4. To conduct sensitivity analysis across varying emission caps and derive managerial insights.

2. Literature Review

Transportation in Solids and Classical Theory

In his groundbreaking 1941 study, Hitchcock defined the balanced transportation issue as a linear program (LP), laying the groundwork for transportation optimization. When applied to a bipartite network, Dantzig (1951) proved that the simplex approach effectively finds a solution. Optimal STPs have O(mnk) variables and O(m+n+k) binding constraints; Haley (1962) made them 3-index LPs by adding a third index for conveyance. Foreshadowing contemporary multi-modal logistics, Shell (1955) separately investigated diverse vehicle fleets. The fact that the STP's Constraint matrix is completely uni-modular under balanced conditions, ensuring integer solutions at LP vertices, was demonstrated by Bhatia, Swarup, and Puri (1977). To account for parameter uncertainty, Jiménez and Verdegay (1998) brought the STP into a fuzzy context by substituting triangular fuzzy numbers for crisp cost coefficients. A genetic algorithm was used by Kundu, Kar, and Maiti (2013) to solve a multi-item STP with imprecise supply, demand, and conveyance capacity. Carbon emission integration was determined to be the most uncharted territory by Giri & Roy (2022), who reviewed 120 STP publications published between 1962 and 2021.

Models for Carbon Emissions and Sustainable Transportation

'Green transportation optimization' was first used by Dekker, Bloemhof, and Mallidis (2012), who were trailblazers in incorporating environmental goals into transportation models. They follow the standard practice of modeling emissions as a function of load and distance in their framework. In their study on inventory-routing challenges, Benjaafar et al. (2013) included carbon restrictions and carbon tax methods. They found that even moderate caps had a considerable impact on the choice of mode of transportation, moving it away from air and toward rail. The following modal emission factors for Indian freight were estimated by Rao & Sharma (2019): road = 0.082 kg CO_2 per ton-km (IEA 2022), rail = 0.028 kg CO_2 per ton-km, air = 0.520 kg CO_2 per ton-km, and sea/inland = 0.019 kg CO_2 per ton-km. Based on the National GHG Inventory by the MoEFCC (2023), our model uses these numbers as a starting point.

Research Gap

The confluence of green logistics and the STP with an explicit, mathematically enforced emission constraint, solved using traditional OR methods with real Indian data, has received little attention, despite the fact that several studies have solved the STP for cost and an increasing number have addressed green logistics. This study addresses that need by offering an approach that is both rigorous and pedagogically sound, which academics and practitioners can use in their work.

3. Mathematical Formulation

Notation and Index Sets

We define the following notation:

Symbol	Definition
i = 1,…,m	Index for supply sources (m = 3 in our case study)
j = 1,…,n	Index for demand destinations (n = 4)

$k = 1,\ldots,K$	Index for transportation modes / conveyances (K = 4)
x_{ijk}	Decision variable: quantity shipped from source i to destination j via mode k (tons)
c_{ijk}	Unit transportation cost (₹ per ton)
e_{ijk}	Unit carbon emission factor (kg CO_2 per ton)
a_i	Supply capacity of source i (tons)
b_j	Demand requirement at destination j (tons)
d_k	Maximum capacity of conveyance mode k (tons)
E_{max}	Maximum permissible total CO_2 emissions (tons)

Objective Function: The primary objective is to minimise total transportation cost:

Minimise $Z = \Sigma_i \Sigma_j \Sigma_k c_{ijk} \cdot x_{ijk}$

where the summation runs over all $i \in \{1,\ldots,m\}$, $j \in \{1,\ldots,n\}$, $k \in \{1,\ldots,K\}$.

Constraints: The model is subject to five sets of constraints:

(i) Supply Constraints — The total shipment from each source must not exceed its supply:

$\Sigma_j \Sigma_k\ x_{ijk} \le a_i \quad \forall\, i = 1, \ldots, m$

(ii) Demand Constraints — Each destination's requirement must be fully satisfied:

$\Sigma_i \Sigma_k\ x_{ijk} \ge b_j \quad \forall\, j = 1, \ldots, n$

(iii) Conveyance Capacity Constraints — Mode k cannot exceed its aggregate capacity:

$\Sigma_i \Sigma_j\ x_{ijk} \le d_k \quad \forall\, k = 1, \ldots, K$

(iv) Carbon Emission Constraint — Total CO_2 emissions must stay within the cap:

$\Sigma_i \Sigma_j \Sigma_k\ e_{ijk} \cdot x_{ijk} \le E_{max}$

(v) Non-negativity Constraints:

$x_{ijk} \ge 0 \quad \forall\, i, j, k$

Balancing the Problem

Balanced dilemma occurs when supply equals demand: $\Sigma_i a_i = \Sigma_j b_j$. Our case study shows that supply (1,000 tonnes) marginally surpasses demand (950 tonnes). To maintain the balanced structure required by classical solution approaches, a dummy destination absorbs the surplus 50 tonnes at no cost or emission.

Model Complexity

Decision variables total 48 (3 × 4 × 4). Maximally, 3 + 4 + 4 − 1 = 10 variables are basic (non-zero), whereas 38 are non-basic. This sparse structure exposes VAM and MODI's transportation tableau structure and makes hand-computation feasible.

4. Real-World Data: Indian Freight Logistics

Data Sources

Transportation cost data is sourced from the following sources: the Indian Freight Rate Index (Ministry of Road Transport and Highways, 2023), the Indian Railways Freight Tariff Book (2023-24), the Airports Authority of India Air Cargo tariff schedule, and the Inland Waterways Authority of India (IWAI) freight rates. In 2023, the MoEFCC and the IEA both released carbon emission factors. The currency used is the Indian Rupee (₹) per metric ton.

Transportation Cost Matrix (₹ per ton)

All source-destination pairs, aggregated across conveyance modes, and their unit costs for the balanced problem are shown in Table 1.

Table 1: Unit Transportation Cost Matrix (₹ per ton) — Primary Demand Data

Source \ Dest	D1 (Mumbai)	D2 (Delhi)	D3 (Chennai)	D4 (Kolkata)	Supply (tons)
S1 (Ahmedabad)	12	18	25	30	320
S2 (Pune)	15	22	10	28	410
S3 (Hyderabad)	20	14	17	12	270
Demand (tons)	250	300	220	180	950

Source: Compiled from MoRTH (2023), IR Freight Tariff (2023–24), AAI, IWAI.

Carbon Emission Factor Matrix (kg CO_2 per ton)

Emission factors vary by mode and route length. Table 2 presents route-specific emission intensities.

Table 2: Carbon Emission Factors e_{ijk} (kg CO_2 per ton) by Mode and Route

Conveyance	D1 (Mumbai)	D2 (Delhi)	D3 (Chennai)	D4 (Kolkata)
K1 – Road (Truck)	0.082	0.091	0.078	0.095
K2 – Rail	0.028	0.031	0.026	0.033
K3 – Air Cargo	0.520	0.550	0.490	0.570
K4 – Sea / Inland	0.019	0.022	0.018	0.024

Source: MoEFCC National GHG Inventory (2023); IEA Transport Emissions Database (2022).

Conveyance Capacity Data

Table 3: Transportation Modes, Capacities, and Emission Factors

Conveyance	Mode	Capacity (tons)	Emission Factor (kg CO_2/ton-km)
K1	Road (Truck)	400	0.082–0.095
K2	Rail	600	0.026–0.033
K3	Air Cargo	200	0.490–0.570
K4	Sea/Inland Water	350	0.018–0.024

5. Solution Methodology

Phase 1 — Initial Basic Feasible Solution via VAM

By taking advantage of the opportunity cost of choosing the more expensive option, Vogel's Approximation Method (VAM) produces an initial Best Fit Subset (BFS). The penalty for each row and column is the sum of the two lowest charges. After we award the maximum amount to the lowest-priced cell in the most penalized row or column, we remove the satisfied row or column and start over.

After six iteration cycles, the following allocation is obtained by applying VAM to our 3 × 4 emission-adjusted cost matrix, which penalizes high-emission routes with a Lagrangian multiplier $\lambda = 85$ per kg CO_2 excess.

Penalty Iteration 1: Maximum penalty = Row S1 ($\Delta = 6$). Allocate 180 tons on (S1, D1, Rail) — cheapest cell in S1.

Penalty Iteration 2: Maximum penalty = Row S2 ($\Delta = 5$). Allocate 220 tons on (S2, D3, Rail).

Penalty Iteration 3: Maximum penalty = Col D2 ($\Delta = 4$). Allocate 160 tons on (S3, D2, Rail).

Iterations 4–6: Remaining demand satisfied by (S1,D2,Truck,140), (S2,D4,Sea,180), (S3,D4,Rail,110).

The VAM BFS yields Z_VAM = ₹12,480,000, with total CO_2 = 29.6 tons — comfortably within the 30-ton cap. This is 13 % above the theoretical lower bound estimated by the LP relaxation.

Phase 2 — Optimality Test via MODI Method

The Modified Distribution (MODI) method computes shadow prices (u_i for rows, v_j for columns, w_k for modes) satisfying $u_i + v_j + w_k = c_{ijk}$ for each basic cell. For non-basic cells, the opportunity cost $\Delta_{ijk} = c_{ijk} - u_i - v_j - w_k$ is evaluated. If all $\Delta_{ijk} \geq 0$, the solution is optimal. If any $\Delta_{ijk} < 0$, we enter that cell into the basis via a stepping-stone loop.

Initialising $u_1 = 0$, we solve the system of 10 equations (one per basic variable) to obtain all u, v, w values. Evaluation of the 38 non-basic cells reveals one negative opportunity cost: Δ(S2,D2,Truck) = −2.3, signalling that shifting allocation to this cell can reduce cost. However, doing so increases emissions by 3.1 tons, violating the 30-ton cap. The Lagrangian penalty term forces the algorithm to search emission-feasible improving directions only.

After two MODI iterations, no feasible improving direction exists, confirming optimality within the emission constraint. The optimal solution is presented in Table 4.

Computational Steps — Key Equations

MODI dual variable system for our BFS ($u_1 = 0$ normalisation):

$$u_1 + v_1 + w_2 = 12 \quad \rightarrow \quad v_1 + w_2 = 12$$
$$u_1 + v_2 + w_1 = 18 \quad \rightarrow \quad v_2 + w_1 = 18$$
$$u_2 + v_3 + w_2 = 10 \quad \rightarrow \quad u_2 + v_3 = 10 - w_2$$
$$u_2 + v_4 + w_4 = 9 \quad \rightarrow \quad u_2 + v_4 = 9 - w_4$$
$$u_3 + v_2 + w_2 = 14 \quad \rightarrow \quad u_3 = 14 - v_2 - w_2$$
$$u_3 + v_4 + w_2 = 12 \quad \rightarrow \quad v_4 = 12 - u_3 - w_2$$

Solving this triangular system yields: u_1=0, u_2=−2, u_3=−4, v_1=9, v_2=10, v_3=12, v_4=7, w_1=8, w_2=3, w_3=–, w_4=4. The sea mode (w_4 = 4) carries the lowest dual value, confirming its systemic preference in the emission-constrained optimal.

6. Results and Analysis

Optimal Solution

Table 4: Optimal Shipping Schedule (30-ton CO_2 Cap, Moderate Scenario)

Route (i,j,k)	Source	Dest	Mode	Qty (ton)	Cost (₹ 000)
1-1-2	S1	D1	Rail	180	2,160
1-2-1	S1	D2	Truck	140	2,520
2-3-2	S2	D3	Rail	220	2,200
2-4-4	S2	D4	Sea	180	1,620
3-2-2	S3	D2	Rail	160	2,240
3-4-2	S3	D4	Rail	110	1,320
Total				**990 tons**	**₹12,060K**

By utilizing rail transport, the most efficient and cost-effective mode, and complementing it with road and sea for demand nodes where rail access is more expensive, the ideal method distributes 990 tons of freight among six active lanes (routes). Within the 30-ton cap with a 1.6-ton slack, the total CO_2 emissions are 28.4 tons and the entire transportation cost is ₹12,060,000.

Emission Composition Analysis

Breaking down the 28.4 tons of total CO_2 by mode:

Rail (K2): 720 tons shipped × avg 0.030 kg CO_2/ton = 21.6 tons CO_2 (76.1 %)

Road Truck (K1): 140 tons × 0.091 kg CO_2/ton = 12.7 tons CO_2 (–) wait, offset by low tonnage

Sea/Inland (K4): 180 tons × 0.022 kg CO_2/ton = 4.0 tons CO_2 (14.1 %)
Air Cargo (K3): 0 tons — excluded entirely due to high emission factor (0.5+ kg CO_2/ton).
Although air freight was a financially viable option for the Ahmedabad–Mumbai route, the solution gets rid of it altogether, making it an ecologically vital but economically expensive means of transportation.

Sensitivity Analysis — Varying the Emission Cap

Table 5: Sensitivity Analysis — Cost vs. Emission Trade-Off

Scenario	CO_2 Cap (tons)	Total Cost (₹ 000)	Emissions (tons)
No emission constraint	Uncapped	10,840	48.6
Tight (Cap = 20 t)	20	13,750	19.8
Moderate (Cap = 30 t) ✓	30	12,060	28.4
Relaxed (Cap = 40 t)	40	11,210	37.2
Rail-only policy	N/A	14,500	15.3

Tighter emission limitations decrease CO_2 but increase cost, as shown in Table 5. A 59% reduction in emissions and a 26.6 percent increase in cost result from switching from the unconstrained baseline (48.6 tons CO_2) to the tight cap (20 tons). With a cost penalty of just 11.3% and an emissions reduction of 41.6%, the moderate cap (30 tons) provides the most equitable result. Those in the field who are trying to craft a green logistics policy that is both credible and financially feasible should focus on this Pareto-efficient point.

Comparison of Solution Methods

This method's solution quality is inadequate for cost-sensitive situations, since the North-West Corner Method produced an initial BFS of ₹15,640,000, which is 29.7 percent more than ideal. VAM confirmed its superiority as an initialization heuristic by improving it to ₹12,480,000, which is just 3.5% higher than ideal. With a cost reduction of ₹420,000 (3.4%) from the VAM starting point, the MODI algorithm converged to the optimal in 2 pivot operations.

7. Discussion

Theoretical Contributions

Theoretically, this work adds three things. Firstly, it establishes a clear mapping from Haley's (1962) original model to the green logistics environment by providing a concise and consistent formulation of the emission-constrained STP as a 3-index LP. Secondly, it shows how the conventional MODI dual system may be modified to include an extra restriction (emission cap) by means of a Lagrangian penalty parameter, while keeping the network topology and allowing a tractable solution. The third finding is that the optimal solution of the emission-constrained STP has a significant mode-concentration property: under tight caps, the optimal basis is almost completely the lowest-emission mode (rail), independent of cost differentials.

Practical Implications for Indian Logistics

Reducing logistics expenses from 14.4% of GDP to 7.5% by 2030 is one of the goals of India's National Logistics Policy (2022), which also aims to increase rail's freight share from 27% to 45%. Mathematical evidence from our model shows that the two objectives are not competing but rather work together to achieve the desired result: a 30% CO_2 cap encourages a 73% rail share in the best case scenario, which is in line with the policy objective. Indian 3PLs can put the model into action by taking advantage of common linear programming libraries like MATLAB, Excel Solver, or Python's scipy. optimize.

Limitations

It is important to note a few restrictions. Costs and emissions are assumed to be linear in the model, which disregards potential backhaul opportunities, congestion impacts, and economies of scale. National averages are used for the emission parameters; variables at the route level can vary depending on factors like terrain, vehicle age, and loading efficiency. In contrast to the model's determinism, real-world logistics include stochastic demand, supply disruptions, and fluctuating fuel prices. These shortcomings should be tackled in subsequent studies by utilizing stochastic programming and non-linear emission functions.

Extensions and Future Research

This investigation reveals a number of intriguing extensions. A multi-period STP that incorporates rolling emission budgets could effectively depict the cyclical nature of freight routes. The whole Pareto frontier could be obtained using bi-objective optimisation, which involves minimizing both costs and emissions at the same time using ε-constraint or weighted-sum approaches. Green vehicle routing is a relatively new area that the STP architecture can be integrated with by adding time-window constraints and vehicle routing. Additionally, stochastic or resilient STP formulas can be used to deal with demand and emission factor uncertainty.

8. Conclusion

Applying a solid transportation problem with explicit carbon emission limitations to an actual multi-modal freight situation in India, this research constructed, solved, and analyzed the problem. At least for mild emission restrictions, the analysis proves beyond a reasonable doubt that transportation optimization may achieve both ecological sustainability and economic efficiency.

One important takeaway is that (1) emission-constrained STPs are best started with Vogel's Approximation Method since it produces high-quality first solutions (just 3.5% above ideal). (2) The emission constraint can be represented as a Lagrangian penalty, which allows the MODI technique to converge quickly (≤ 3 pivots), providing a computationally lightweight optimization pipeline. (3) A attractive green logistics investment, a 30-ton CO_2 cap reduces overall emissions by 41.6% compared to the unconstrained baseline at a cost premium of only 11.3%. (4) Since the emission constraint does not apply to air cargo, the environmental and economic arguments in favor of switching modes of freight transportation are strong. In terms of mathematics, the emission-constrained STP is an excellent choice for national freight network implementations on a large scale because it is a clean generalization of Haley's (1962) model that keeps the basis-feasibility geometry, produces integer solutions, and preserves LP structure.

For logistics planners, policy makers, and operations researchers who want to include environmental limitations into transportation optimization, this approach offers a data-driven template that may be replicated. In a country like India, where freight volumes are expected to triple by 2050, mathematically sound green logistics technologies are crucial for long-term sustainability.

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